However, the opacity could remain constant or increase over this energy range if the scattering amplitude had a small real part. The absence of any significant change in the shape of the diffraction peak would then require either that the effective radius parameters of $\mathrm{Re}H(b,s)$ and $\mathrm{Im}H(b,s)$ be nearly the same or that both change in a correlated fashion. However, the essentially complete absence of any structure in the diffraction peak,[32] and, to a lesser extent, the relatively large uncertainties in the total cross section, prevent more refined analysis. Without more detailed knowledge about the scattering amplitude, it appears unlikely that measurements of the elastic π-p cross section in the diffraction-scattering region can provide more than a rough consistency check for dynamical calculations. The same will of course be true of p-p scattering if the diffraction peak ultimately ceases to shrink, as suggested in the preceding sections. Experiments designed to detect any real part of the scattering amplitudes would consequently be of great interest.

[32] There may be some evidence of structure and shrinkage in the high $|t|$ region in the elastic $\pi^- - p$ cross sections at lower energies [Ref. 2]. It would be interesting to investigate this point in more detail.

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Pion-Proton Diffraction Scattering at Very Small Momentum Transfers*

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The model of high-energy scattering based on a moving pole and a fixed cut in the angular-momentum plane is applied to π-p scattering at very small momentum transfers, and is found to agree with the experiments which give a departure from an exponential diffraction peak.

IN a recent paper,[1] we have proposed a model of high-energy scattering, applicable to all strongly interacting particles, which is based on the assumption that the dominant singularities in the angular momentum plane are a Pomeranchuk pole and a fixed branch cut. For large values of the energy $(\surd s)$ and small momentum transfers $[\surd(-t)]$ the cross section is given asymptotically by[2,3]

$$d\sigma/dt=(\pi/M_1{}^2M_2{}^2)\,|g(t)w^{\alpha_P(t)-1}+f(t)/\ln w|^2 \quad (1)$$

and

$$\sigma_T=(4\pi/M_1M_2)[g(0)+f(0)/\ln w],$$

where $w=(s/s_0)$, the dominant branch point $\alpha_2(t)=1$ for all t, and M_1, M_2 are the masses of the colliding particles. In the pole-fixed-cut model the normalization constant s_0 acquires special significance and taking $s_0=2M_1M_2$, this model explains both $p+p$ and π^-+p scattering data.[2,4,5] In the case of $p+p$ scattering w is relatively small in the range 7–30 BeV/c and the Pomeranchuk pole term in (1) dominates leading to a shrinkage of the diffraction peak, as given by a simple Regge pole. On the other hand, for π^-+p scattering between 7–17 BeV/c, w is several times larger and, for t not too close to zero, $w^{\alpha_P(t)-1}$ will be very small suppressing the pole term; the fixed cut then takes over and this gives a nonshrinking diffraction pattern, as is observed in π^-+p scattering. However, for t very small the pole should make a significant contribution and evidence of this is important because it establishes the presence of more than one significant singularity in the α plane.

Some evidence to this effect has recently been obtained from $\pi-p$ elastic scattering giving the differential cross section for very small values of t.[6] The differential cross section at these small momentum transfers deviates from the exponential shape usually adopted, exhibiting an upward curvature. This result indicates that there is an additional contribution at very small t over and above the fixed-cut contribution. Thus, the presence of at least two significant singu-

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[3] The asymptotic form of the branch cut contribution was obtained in Ref. (1) by integrating by parts and dropping terms of higher order in $(1/\ln s)$. This calculation is exact for a discontinuity of the form $f(t,\alpha)=\theta(\alpha_2-\alpha)$. Such a discontinuity is, of course, unphysical, and in a less phenomenological treatment one might obtain $f(t)/(\ln s)^\beta$, say, with $\beta>1$ and noninteger. Due care should then be taken to avoid fixed cuts which cause the amplitude to become unbounded. However, the approximate analyses we have made are clearly insensitive to such a change, and the experimental features remain unaltered.

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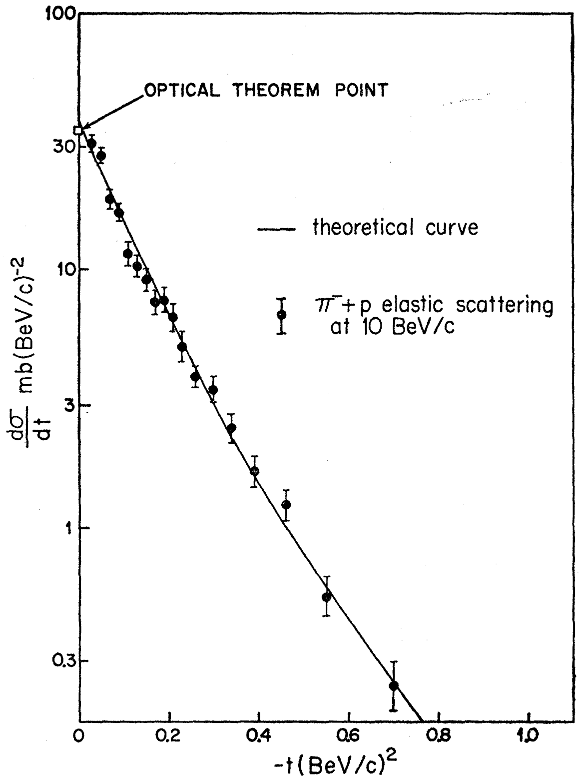

FIG. 1. The experimental points for $\pi^- + p$ scattering at 10 BeV/c are shown together with the theoretical curve obtained from a least-squares fit assuming $f(0)/g(0) = 2.5$.

larities in the α plane is strongly indicated by the qualitative features of the data.

Brandt *et al.*[6] have already remarked that their data differ significantly from a simple exponential peak and they obtained a fit to the data using different exponentials in the $t < -0.16$ and $t > -0.16$ regions. We wish to fit the data by a sum of exponentials obtained from (1). Because the curve for larger $|t|$ is exponential,[2] we put $f(t) = f(0)e^{\gamma t}$, and as the pole only contributes significantly for very small t, we may set $g(t) = g(0)e^{\rho t}$ and $\alpha_p(t) = 1 + \dot{\alpha}_p(0)t$. From Eq. (1), we get

$$\frac{d\sigma}{dt} = \frac{\pi}{M_1^2 M_2^2} \left| g(0)e^{\tau t} + \frac{f(0)}{\ln w} e^{\gamma t} \right|^2, \qquad (3)$$

where $\tau = \rho + \dot{\alpha}_p \ln w$. Total cross-section data gives[2] $f(0)/g(0) \sim 2.5$ and, using this value, the scattering at 10 Bev/c gives a least-squares fit to the data for $-1.0 < t < -0.02$ with

$$g(0) = 0.47, \qquad f(0) = 0.28 \ln w = 1.18,$$
$$\tau = 6.4\ (\text{BeV})^{-2}, \qquad \gamma = 2.4\ (\text{BeV})^{-2}.$$

The value of ρ and $\dot{\alpha}_p(0)$ cannot be determined separately from this experiment, but if we assume that $\dot{\alpha}_p(0) = 1$, we have $\rho = 2.1$. The theoretical curve and the experimental data are shown in Fig. 1.

There is a 10% difference between the optical theorem point and the extrapolated value of $d\sigma/dt$ at $t = 0$. This difference is of the order of magnitude of the over-all experimental error for small t and means that $\text{Re}A(s,t)$ is about 30% of the total amplitude. Our present model is based on the assumption that $\text{Re}A(s,t) = 0$, but our phenomenological model should tolerate a small real part at these accelerator energies, because the effect on $d\sigma/dt$ is negligible.

A Fisher F test gives a 99.9% chance that the pole + cut model is significantly better than the Regge one-pole model.[7] The experimental points exhibit a systematic deviation from a straight line, which is explained qualitatively by the competition of the Pomeranchuk pole and the cut as t approaches zero.

In the present work, we have fixed the ratio $f(0)/g(0) = 2.5$ in accordance with our previous fit to the $\pi^\pm + p$ total cross-section data,[2,8] and we obtain individual results for $f(0)$ and $g(0)$, given above, which are the same as the earlier results.[2] The slopes τ and γ obtained from our earlier analysis of the Foley *et al.* differential cross-section data[2] are $\tau = 5.2(\text{BeV})^{-2}$ and $\gamma = 3.0(\text{BeV})^{-2}$, which agree to within 25% with the τ and γ quoted above.

Thus, our model fits all the presently available data on $\pi^\pm + p$ and $p + p$ scattering. It also appears to agree with the results on $K^\pm + p$ scattering, which give diffraction peaks exhibiting little or no shrinkage,[9] although further analysis must be carried out to verify this result.

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